Relations among Elements of the Quark Mass Matrices

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Abstract

Current experimental data suggest some relations between the Kobayashi-Maskawa Matrix and the quark mass ratios, namely $|V_{us}| \sim \sqrt{m_d/m_s}$, $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$ and $|V_{cb}| \sim m_s/m_b$. We consider these relations seriously in this paper. As a result, they offer us some attractive relations among the elements of the mass matrices and bring us a problem. They might give us informations about the origin of Yukawa couplings.

1 Introduction

It is very interesting to consider the quark mass matrices and the Kobayashi-Maskawa Matrix (KM) matrix [1]. It is because the origin of the quark masses and the KM matrix is not understood in the standard model (SM), though the SM is able to explain many experimentations. The fundamental theory is expected to explain the relations among the parameters in the SM. For that reason, understanding of relations among the SM parameters would offer us important information about the fundamental theory.

Current experimental data [2] give us some hints for relations between the KM Matrix and the quark mass ratios, namely $|V_{us}| \sim \sqrt{m_d/m_s}$, $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$ and $|V_{cb}| \sim m_s/m_b$. These relations might be just an accident.

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But they would be important informations about the origin of Yukawa couplings, therefore, it is meaningful to stand on such a point of view. Starting with a specific theory or symmetry, many attempt have been made to derive relations between the KM matrix and the quark mass ratios using the specific mass matrices with less than 10 parameters [3]-[12]. However, we start from the KM matrix and the quark masses using generic and minimal parametrization of the mass matrices. This approach has the advantage of considering the origin of quark masses and the flavor mixing.

Consequently, we can bring the relations between elements of the KM matrix and the quark mass ratios back to relations among elements of the mass matrices. It is not just relations among parameters but relations among elements of the mass matrices, so it is very attractive because they might give suggestions about the origin of Yukawa couplings.

In addition, they give us a problem which can not solve by only flavor symmetry. This is common problem when we suppose some natural conditions. Under the conditions, we must introduce other mechanism or something.

This paper is organized as follows. In section 2, we make our starting point about the KM matrix clear. In section 3, we introduce one set of minimally generic quark mass matrices and parametrize the elements of them. In section 4, we consider the relations among the elements of the quark mass matrices. Finally, in section 5, we summarize the results.

2 The Kobayashi-Maskawa matrix

Current experimental data provide us hints of the relations between quark mass ratios and the KM matrix elements [2]:

$$\begin{aligned} |V_{us}| &\sim \sqrt{\frac{m_d}{m_s}} \sim 0.22, \\ |V_{cb}| &\sim \frac{m_s}{m_b} \sim 0.03 \text{ to } 0.05, \\ \left|\frac{V_{ub}}{V_{cb}}\right| &\sim \sqrt{\frac{m_u}{m_c}} \sim 0.06 \text{ to } 0.10. \end{aligned} \tag{1}$$

These relations may be just an accident, but we consider the relations seriously.

The up (down) part mass matrix $M_{\rm u}$ ($M_{\rm d}$) is diagonalized by unitary matrices $U_{\rm u}$ and $V_{\rm u}$ ($U_{\rm d}$ and $V_{\rm d}$),

$$U_{\rm u}^\dagger M_{\rm u} V_{\rm u} = D_{\rm u}, \qquad U_{\rm d}^\dagger M_{\rm d} V_{\rm d} = D_{\rm d}, \tag{2}$$

where $D_{\rm u,d}$ are diagonal matrices. Then the KM matrix $V_{\rm KM}$ is

$$V_{\rm KM} = U_{\rm u}^\dagger U_{\rm d}. \tag{3}$$

The KM matrix is almost the unit matrix, especially $|V_{ub}|$ and $|V_{td}|$ are very small in comparison with the other elements. So let us suppose that $U_{\rm u}$ and $U_{\rm d}$ are almost the unit matrix, and that (1,3) and (3,1) elements of $U_{\rm u,d}$ are very small. If we do not suppose them, we need large cancellations between up and down part to obtain realistic KM matrix. But, now, we suppose there is no such cancellation. Then the relations (1) remind us that the KM matrix is expressed by quark mass ratios as follows:

$$\begin{aligned}
V_{\rm KM} &= U_{\rm u}^\dagger U_{\rm d}, \\
U_{\rm u} &\sim \begin{pmatrix} \exp(\mathrm{i}\theta) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{\dfrac{m_u}{m_c}} & 0 \\ -\sqrt{\dfrac{m_u}{m_c}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
U_{\rm d} &\sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \dfrac{m_s}{m_b} \\ 0 & -\dfrac{m_s}{m_b} & 1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{\dfrac{m_d}{m_s}} & 0 \\ -\sqrt{\dfrac{m_d}{m_s}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\end{aligned} \tag{4}$$

On this point of view, we consider the quark mass matrices in this paper.

3 Quark mass matrices

If we assume up and down part quark mass matrices are hermitian, the expression (4) can be constructed when the up and down part mass matrices

M_u and M_d have the following form:

$$
\begin{aligned}
M_\mathrm{u} &= \begin{pmatrix} a_\mathrm{u} & d_\mathrm{u}\exp(\mathrm{i}\theta) & 0 \\ d_\mathrm{u}\exp(-\mathrm{i}\theta) & b_\mathrm{u} & 0 \\ 0 & 0 & c_\mathrm{u} \end{pmatrix}, \\
M_\mathrm{d} &= \begin{pmatrix} a_\mathrm{d} & d_\mathrm{d} & 0 \\ d_\mathrm{d} & b_\mathrm{d} & e_\mathrm{d} \\ 0 & e_\mathrm{d} & c_\mathrm{d} \end{pmatrix}, \qquad (5)
\end{aligned}
$$

where $a_\mathrm{u} \sim d_\mathrm{u}$, $a_\mathrm{d} \sim e_\mathrm{d}$ and θ are real. These mass matrices lead (1,2) mixing for up part and (1,2) and (2,3) mixing for down part, so they are suitable to (4). The above quark mass matrices are minimally generic form of the SM, that is, there are just 10 parameters and there is no unmeasurable parameter.

Starting from the above form (5), we do not lose any generalities, because arbitrary quark mass matrices can be transformed into the above form. Arbitrary up and down part quark mass matrices $\hat{M}_\mathrm{u,d}$ can be transformed into hermitian matrices $\hat{H}_\mathrm{u,d}$ without changing any physical contents. So if arbitrary hermitian quark mass matrices can be transformed into the above form (5) without changing any physical contents, it means that any quark mass matrices can be transformed into the form (5) and that the form does not lose any generalities. The transformation is the following:

$$
\mathcal{U}^\dagger \hat{H}_\mathrm{u,d} \mathcal{U} = M_\mathrm{u,d}, \qquad (6)
$$

where $\mathcal{U}$ is the unitary matrix which is determined in the following way. At first, $\mathcal{U}_{i3}$ $(i = 1 \sim 3)$ is determined to be one of eigenvectors of $\hat{H}_\mathrm{u}$. Then we can obtain

$$
M_{\mathrm{u}\ 13} = M_{\mathrm{u}\ 23} = M_{\mathrm{u}\ 31} = M_{\mathrm{u}\ 32} = 0 \qquad (7)
$$

with the help of the unitarity of $\mathcal{U}$. Next, $\mathcal{U}_{i1}$ is determined by satisfying the equations

$$
\begin{aligned}
\mathcal{U}^*_{i3} \hat{H}_{\mathrm{d}\ ij} \mathcal{U}_{j1} &= 0, \qquad (8) \\
\mathcal{U}^*_{i3} \mathcal{U}_{i1} &= 0, \qquad (9)
\end{aligned}
$$

where $i, j = 1 \sim 3$. (8) guarantees

$$
M_{\mathrm{d}\ 13} = M_{\mathrm{d}\ 31} = 0. \qquad (10)
$$

Then $\mathcal{U}_{i2}$ is determined automatically because of unitarity of $\mathcal{U}$. Since $\mathcal{U}$ is unitary matrix, quark masses do not depend on the choice of $\mathcal{U}$. Moreover, the measurements of the KM matrix do not change, because both up and down part are transformed by the same unitary matrix $\mathcal{U}$.

The mass matrices (5) are minimally generic one, so we can express $a_{\rm u} \sim d_{\rm u}$, $a_{\rm d} \sim e_{\rm d}$ and θ in terms of physical contents. We can choose $\hat{H}_{\rm u,d}$ as

$$\begin{aligned} \hat{H}_{\rm u} &= \begin{pmatrix} m_{\rm u1} & 0 & 0 \\ 0 & m_{\rm u2} & 0 \\ 0 & 0 & m_{\rm u3} \end{pmatrix}, \\ \hat{H}_{\rm d} &= V_{\rm KM} \begin{pmatrix} m_{\rm d1} & 0 & 0 \\ 0 & m_{\rm d2} & 0 \\ 0 & 0 & m_{\rm d3} \end{pmatrix} V_{\rm KM}^{\dagger}, \end{aligned} \tag{11}$$

where $V_{\rm KM}$ is the KM matrix and $|m_{\rm u(d)i}|$ ($i = 1 \sim 3$) is the i-th generation up (down) part quark mass. These mass matrices produce quark masses and the KM matrix accurately. Starting from (11) and using the Wolfenstein form [13] as the KM matrix:

$$V_{\rm Wolfenstein} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - {\rm i}\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-{\rm i}\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{12}$$

we can obtain

$$\mathcal{U} \simeq \begin{pmatrix} \exp({\rm i}\phi_1) & \lambda\sqrt{\rho^2+\eta^2}\exp(-{\rm i}\phi_3) & 0 \\ -\lambda\sqrt{\rho^2+\eta^2}\exp\{{\rm i}(\phi_1+\phi_3)\} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned} \phi_1 &= \arg(1-\rho+{\rm i}\eta), \\ \phi_3 &= \arg(\rho+{\rm i}\eta). \end{aligned} \tag{14}$$

Then we can obtain the leading terms of $a_{\rm u} \sim d_{\rm u}$, $a_{\rm d} \sim e_{\rm d}$ and θ:

$$a_{\rm u} \simeq m_{\rm u1} + m_{\rm u2}\lambda^2(\rho^2+\eta^2),$$

$$
\begin{aligned}
b_{\rm u} &\simeq m_{\rm u2}, \\
c_{\rm u} &= m_{\rm u3}, \\
d_{\rm u} &\simeq m_{\rm u2}\lambda(\rho^2+\eta^2), \\
a_{\rm d} &\simeq m_{\rm d1}+m_{\rm d2}\lambda^2|1-\rho-{\rm i}\eta|^2, \\
b_{\rm d} &\simeq m_{\rm d2}, \\
c_{\rm d} &\simeq m_{\rm d3}, \\
d_{\rm d} &\simeq m_{\rm d2}\lambda|1-\rho-{\rm i}\eta|, \\
e_{\rm d} &\simeq m_{\rm d3}A\lambda^2, \\
\theta &\simeq \arg(-\rho+{\rm i}\eta)-\arg(1-\rho+{\rm i}\eta)\,.
\end{aligned}
\tag{15}
$$

If we take

$$
\begin{aligned}
&m_{\rm u1}=-m_u, \quad m_{\rm u2}=m_c, \quad m_{\rm u3}=m_t, \\
&m_{\rm d1}=-m_d, \quad m_{\rm d2}=m_s, \quad m_{\rm d3}=m_b,
\end{aligned}
\tag{16}
$$

where m_u, m_d, m_c, m_s m_t and m_b are the up, down, charm, strange, top and bottom quark masses, respectively, and make a certain phase rotation, we can obtain the quark mass matrices $M_{\rm u,d}$ as

$$
\begin{aligned}
M_{\rm u} &\simeq \begin{pmatrix} -m_u+m_c\lambda^2(\rho^2+\eta^2) & m_c\lambda(-\rho+{\rm i}\eta) & 0 \\ m_c\lambda(-\rho-{\rm i}\eta) & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \\
M_{\rm d} &\simeq \begin{pmatrix} -m_d+m_s\lambda^2|1-\rho-{\rm i}\eta|^2 & m_s\lambda(1-\rho+{\rm i}\eta) & 0 \\ m_s\lambda(1-\rho-{\rm i}\eta) & m_s & m_bA\lambda^2 \\ 0 & m_bA\lambda^2 & m_b \end{pmatrix}.
\end{aligned}
\tag{17}
$$

To see the above matrices (17) more simply, let us introduce three parameters related to the measurements of the KM matrix, p_1, p_2 and p_3:

$$
\begin{aligned}
p_1 &= \frac{m_c\lambda^2(\rho^2+\eta^2)}{m_u}, \\
p_2 &= \frac{m_s\lambda^2|1-\rho-{\rm i}\eta|^2}{m_d}, \\
p_3 &= \frac{m_bA\lambda^2}{m_s}.
\end{aligned}
\tag{18}
$$

Using these parameters and making a certain phase rotation, we can write the matrices (17) as

$$
\begin{aligned}
M_{\mathrm{u}} &\simeq \begin{pmatrix} (p_1-1)m_u & \sqrt{p_1 m_u m_c}\exp(\mathrm{i}\theta) & 0 \\ \sqrt{p_1 m_u m_c}\exp(-\mathrm{i}\theta) & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \\
M_{\mathrm{d}} &\simeq \begin{pmatrix} (p_2-1)m_d & \sqrt{p_2 m_d m_s} & 0 \\ \sqrt{p_2 m_d m_s} & m_s & p_3 m_s \\ 0 & p_3 m_s & m_b \end{pmatrix},
\end{aligned} \tag{19}
$$

where

$$
\theta \simeq \arg(-\rho + \mathrm{i}\eta) - \arg(1-\rho+\mathrm{i}\eta) \tag{20}
$$

and $p_1 \sim p_3$ and θ characterize the KM matrix. At this time, we can obtain

$$
\begin{aligned}
|V_{us}| &\simeq \left| \sqrt{\frac{p_2 m_d}{m_s}} - \sqrt{\frac{p_1 m_u}{m_c}} \exp(\mathrm{i}\theta) \right|, \\
|V_{cb}| &\simeq \frac{p_3 m_s}{m_b}, \\
\left|\frac{V_{ub}}{V_{cb}}\right| &\simeq \sqrt{\frac{p_1 m_u}{m_c}}, \\
\left|\frac{V_{td}}{V_{ts}}\right| &\simeq \sqrt{\frac{p_2 m_d}{m_s}}.
\end{aligned} \tag{21}
$$

4 Discussion

Using current experimental data, we can find that three parameters $p_1 \sim p_3$ are nearly equal to 1. If we take current experimental data [14, 2]

$$
\begin{aligned}
\frac{m_u}{m_c} &= 4.26 \times 10^{-3} \times (1 \pm 0.196), \\
\frac{m_d}{m_s} &= 4.97 \times 10^{-2} \times (1 \pm 0.045), \\
\frac{m_s}{m_b} &= 3.35 \times 10^{-2} \times (1 \pm 0.144),
\end{aligned} \tag{22}
$$

and

$$
\lambda = 0.217 \sim 0.224,
$$

$$\begin{aligned}\lambda\sqrt{\rho^2+\eta^2} &= 0.06 \sim 0.10, \\ A\lambda^2 &= 0.035 \sim 0.042, \end{aligned} \tag{23}$$

then we can obtain

$$\begin{aligned} p_1 &= \frac{m_c\lambda^2(\rho^2+\eta^2)}{m_u} = 0.71 \sim 2.92, \\ p_2 &= \frac{m_s\lambda^2|1-\rho-\mathrm{i}\eta|^2}{m_d} = 0.71 \sim 1.27, \\ p_3 &= \frac{m_b A\lambda^2}{m_s} = 0.91 \sim 1.46. \end{aligned} \tag{24}$$

It is instructive to consider energy dependence of the above parameters $p_1 \sim p_3$. The parameters p_1 and p_2 in (18) are almost energy independent, because ρ, η, λ, m_u/m_c and m_d/m_s are almost energy independent. As for the parameter p_3, A has energy dependence, but m_s/m_b has almost the same energy dependence in the case that $\tan\beta$ has magnitude of $O(1)$ [15]. Then the parameter p_3 is energy independent in the case of $\tan\beta \sim O(1)$. Hence the parameters which are estimated at the weak scale are meaningful enough.

The parameters $p_1 \sim p_3$ are nearly equal to 1. It means that the relations between the KM matrix elements and the quark mass ratios are understood in terms of the elements of the quark mass matrices. It is very interesting, because we might be able to regard the simple relations among the elements as the relations among the original Yukawa couplings. When p_1, p_2 equal to 1, we obtain

$$M_{\mathrm{u}\ 11} \simeq 0, \qquad M_{\mathrm{d}\ 11} \simeq 0 \tag{25}$$

and

$$\begin{aligned} |V_{us}| &\simeq \left|\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\exp(\mathrm{i}\theta)\right|, \\ \left|\frac{V_{ub}}{V_{cb}}\right| &\simeq \sqrt{\frac{m_u}{m_c}}, \\ \left|\frac{V_{td}}{V_{ts}}\right| &\simeq \sqrt{\frac{m_d}{m_s}}. \end{aligned} \tag{26}$$

And then we obtain

$$\arg(\rho+\mathrm{i}\eta) \simeq \frac{\pi}{2}, \tag{27}$$

taking account of

$$\sqrt{\frac{m_d}{m_s}} \simeq \lambda \gg \sqrt{\frac{m_u}{m_c}} \gg \left| \sqrt{\frac{m_d}{m_s}} - \lambda \right|. \tag{28}$$

This point is similar to the Fritzsch *Ansatz* case [3]. Moreover, when p_3 equals to 1, we obtain

$$M_{\mathrm{d}\ 22} \simeq M_{\mathrm{d}\ 23}\ (M_{\mathrm{d}\ 32}) \tag{29}$$

and

$$|V_{cb}| \simeq \frac{m_s}{m_b}. \tag{30}$$

If all the parameters p_1, p_2 and p_3 are equal to 1, the mass matrices (19) become exceedingly simple form:

$$\begin{aligned} M_{\mathrm{u}} &= \begin{pmatrix} 0 & d_{\mathrm{u}} \exp(\mathrm{i}\theta) & 0 \\ d_{\mathrm{u}} \exp(-\mathrm{i}\theta) & b_{\mathrm{u}} & 0 \\ 0 & 0 & c_{\mathrm{u}} \end{pmatrix}, \\ M_{\mathrm{d}} &= \begin{pmatrix} 0 & d_{\mathrm{d}} & 0 \\ d_{\mathrm{d}} & b_{\mathrm{d}} & b_{\mathrm{d}} \\ 0 & b_{\mathrm{d}} & c_{\mathrm{d}} \end{pmatrix}. \end{aligned} \tag{31}$$

They have only 7 parameters, that is 6 hierarchical magnitudes of elements and 1 phase. These form might offer us the origin of the quark mass matrices.

The top quark mass is very large, so (2,2) element and (2,3) element ((3,2) element) of the up part mass matrices might also take the same value. In this case, we obtain V_{cb} as

$$|V_{cb}| \simeq \frac{m_s}{m_b} \pm \frac{m_c}{m_t}\ . \tag{32}$$

Note that the quark mass matrices which Fritzsch *et al.* have assumed in Ref. [10] have different situation in V_{cb}. They start from "democratic mass matrix", that is, rank 1 matrix, and add a cirtain small parameters to it, then the mass matrices lead to

$$|V_{cb}| \simeq \frac{1}{\sqrt{2}} \left(\frac{m_s}{m_b} \pm \frac{m_c}{m_t} \right). \tag{33}$$

In this case, the parameter p_3 is not nearly equal to 1 but nearly equal to $1/\sqrt{2}$ because $m_c/m_t \ll m_s/m_b$.

P. Ramond *et al.* have suggested the similar structure mass matrices to the above form (31) in Ref. [16]. But they do not have mentioned the possibility that the (2,2) element and (2,3) (or (3,2)) element of down part mass matrices are almost the same magnitudes each other in Ref. [16].

To explain $M_{\mathrm{d}\ 22} \simeq M_{\mathrm{d}\ 23}(M_{\mathrm{d}\ 32})$ is a problem, even if we explain zero elements by flavor symmetry. If we regard the origins of $M_{\mathrm{d}\ 22}$ and $M_{\mathrm{d}\ 23}$ ($M_{\mathrm{d}\ 32}$) as the same one Higgs vacuum expectation value and Yukawa coupling, and if the origin of $M_{\mathrm{Simpler\ d}\ 33}$ is different from it, it is difficult to explain them by giving charges to fields. Namely, $d_{\mathrm{L}\ 2}$ (down part left handed quark field of the 2nd family) and $d_{\mathrm{L}\ 3}$ (down part left handed quark field of the 3rd family) must have the same charge to couple with the same Higgs. Similarly, $d_{\mathrm{R}\ 2}$ (down part right handed quark field of the 2nd family) and $d_{\mathrm{R}\ 3}$ (down part right handed quark field of the 3rd family) must do so. On the other hand, $d_{\mathrm{L}\ 3}$ and $d_{\mathrm{R}\ 3}$ couple with other Higgs and it lead large (3,3) element of the down part mass matrix. At this time, (2,2), (2,3) and (3,2) elements of the down part mass matrix must be as large as (3,3) element of the down part mass matrix because $d_{\mathrm{L}\ 2}$ and $d_{\mathrm{L}\ 3}$ have the same charge and so do $d_{\mathrm{R}\ 2}$ and $d_{\mathrm{R}\ 3}$. This is caused by the same magnitude elements in the same column or row.

This situation does not change when we assume that quark mass matrices have only 6 hierarchical elements except phases and that $|V_{cb}| \simeq m_s/m_b$ mainly comes from the second and the third columns and rows of down part mass matrix. If we suppose there is no same order element in the same column and the same row, only $M_{\mathrm{d}\ 33}$ must be the largest to obtain different order quark masses. Then we obtain $M_{\mathrm{d}\ 33} \simeq m_b$ from trace of $M_{\mathrm{d}} M_{\mathrm{d}}^\dagger$. So we can write the (2,3) part of M_{d} as

$$M \simeq \begin{pmatrix} x & y \\ z & m_b \end{pmatrix} , \tag{34}$$

where $x, y, z \ll m_b$. (We assume x, y and z are real for simplicity.) At this time,

$$MM^{\mathrm{T}} \simeq \begin{pmatrix} x^2 + y^2 & ym_b \\ ym_b & m_b^2 \end{pmatrix} \tag{35}$$

must be almost diagonalized by

$$\begin{pmatrix} 1 & \dfrac{m_s}{m_b} \\ -\dfrac{m_s}{m_b} & 1 \end{pmatrix} \tag{36}$$

in order to mainly obtain $|V_{cb}| \simeq m_s/m_b$ from the (2,3) part of M_d. Hence we obtain $y \simeq m_s$. Then determinant of M is

$$xm_b - zm_s \simeq m_s m_b \ . \tag{37}$$

So we obtain $x \simeq m_s$, taking account of $z \ll m_b$. However, this contradicts the supposition that there is no same order elements in the same column and the same row. Therefore, we need the same order elements at the same column or row in (2,3) part of M_d when we assume that $|V_{cb}| \simeq m_s/m_b$ mainly comes from (2,3) part of M_d and that only 6 hierarchical elements of quark mass matrices except phases. This is the very reason that the Fritzsch *Ansatz* [3] can not explain $|V_{cb}|$ and that the Branco-Silva-type [9] can explain $|V_{cb}|$. Anyway, in order to explain $p_3 \simeq 1$, we need a mechanism or something.

5 Summary

We have studied relations between the quark masses and the KM matrix elements on the minimally generic quark mass matrices. They are hermitian matrices and have 3 texture zeros. Furthermore, they have only 10 parameters, so we can express all their elements in terms of only physical contents. Then we have seen that current experimental data suggest some interesting simple relations among the elements in the minimally generic form. In order to clear this point, we have introduced three parameters which are almost energy independent for $\tan\beta \sim O(1)$. The relations among the elements might offer us the relations among the original Yukawa couplings, so it is very interesting. In addition, we have seen the possibility that the relations $|V_{us}| \sim \sqrt{m_d/m_s}$, $|V_{cb}| \sim m_s/m_b$ and $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$ are explained by only 6 hierarchical elements of mass matrices and 1 phase.

Moreover, it has pointed out that to explain the relation $|V_{cb}| \sim m_s/m_b$ has difficulty in flavor symmetry if we assume that the quark mass matrices have only 6 hierarchical elements except phases and that $|V_{cb}| \simeq m_s/m_b$

mainly comes from down part. This situation does not change in general. So, in order to explain $|V_{cb}| \simeq m_s/m_b$, we might have to introduce a mechanism or something.

In the near future, the three parameters will be made out more accurately in detailed experiments, for example experiments in b-factory. They will offer us important information about the origin of the quark masses and the KM matrix.

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